Example 4: Ascertain the size of the sample from the following particulars:
Standard deviation of population $\sigma_p = 4$

Mean of population $\mu = 24$
Mean of sample or $\overline{X}_s = 22$ and

Level of confidence = 99%
(Z value at 99% = 2.5758)

Solution: Given, Standard deviation of population $(\sigma_p) = 4$, Mean of population $(\mu) = 24$, Mean of sample $(\overline{X}_s) = 22$ and $Z = 2.5758$.

$$\text{Since, } \sigma_{\bar{x}} = \frac{\sigma_p}{\sqrt{n}} \qquad \Rightarrow \sigma_{\bar{x}} = \frac{4}{\sqrt{n}}$$

$$\because \; Z = \frac{\overline{X}_s - \mu}{\sigma_{\bar{x}}} = \frac{22-24}{4/\sqrt{n}}$$

$$\Rightarrow 2.5758 = \frac{-2}{4/\sqrt{n}}; \qquad \Rightarrow 2.5758 \times 4 = -2\sqrt{n}$$

$$\Rightarrow \sqrt{n} = -5.1516; \qquad \Rightarrow n = 26.53 \approx 27$$

Determining Sample Size (n) When Estimating the Population Proportion

In the estimation of population proportion (p) sample proportion $\hat{p}$ is used for a sample of size n.

The standard error of $(\hat{p}) = \sqrt{\frac{pq}{n}}$ (p is known).

For large samples,

$$Z = \frac{\hat{p} - p}{\text{S.E.}(\hat{p})} = \frac{E}{\sqrt{\frac{pq}{n}}}$$

Where, $E = \hat{p} - p$ (sampling error)

$$\sqrt{n} = \frac{Z\sqrt{pq}}{E} \qquad \text{Or } n = \frac{Z^2 pq}{E^2}$$

Example 5: Suppose p is estimated as 60 % and the confidence level is set at 95%. If the allowable error in estimating the population proportion is not to be greater than 3%, calculate the required sample size.

Solution: At 95% confidence level Z = 1.96.

The sample size is given by $\sqrt{n} = \frac{Z\sqrt{pq}}{E}$

Here p = 0.6, q = 0.4, E = 0.03

$$\text{So } \sqrt{n} = \frac{1.96\sqrt{0.6 \times 0.4}}{0.03} = \frac{1.96\sqrt{0.24}}{0.03}$$

$$n = \frac{1.96 \times 1.96 \times 0.24}{0.0009} = \frac{0.921984}{0.0009} = 1024.43 \approx 1024$$

Example 6: Management of Hotel Bandhan, Lucknow is interested in determining the percentage of the hotel's guests who stay for more than 3 days. The reservation manager wants to be 95 per cent confident that the percentage has been estimated to be within ± 3% of the true value. What is the most conservative sample size needed for this problem? Value of $p = 0.5$.

$Z = 1.96$ (as per table of area under normal curve for the given confidence level of 95%).

Solution: We have been given the following:
Population is infinite
$E = 0.03$ (since the estimate should be within 3% of the true value)
$Z = 1.96$ (as per table of area under normal curve for the given confidence level of 95%)

As we want the most conservative sample size we shall take the value of $p = 0.5$ and $q = 0.5$. Using all this information, we can determine the sample size for the given problem as under:

$$n = \frac{Z^2 pq}{E^2} = \frac{(1.96)^2 \cdot (0.5)(1-0.5)}{(0.03)^2} = \frac{0.9604}{0.0009} = 1067.11 \simeq 1067$$

Thus, the most conservative sample size needed for the problem is = 1067

Example 7: Determine the sample size necessary to estimate a population proportion to be within 0.03 with 99% confidence, assuming you have no knowledge of the approximate value of the sample proportion.

Solution: For the 99% level of confidence, the $Z = 1.96$
Error of Estimation (E) = 0.03
Assumes, if the population proportion is not more than 0.5 then $p = 0.5$ and $q = 0.5$. So,

$$\text{Sample Size (n)} = \frac{Z^2 pq}{E^2}$$

$$= \frac{(1.96)^2\, 0.5 \times (1-0.5)}{(0.03)^2} = \frac{0.9604}{0.0009} = 1067.1 \approx 1067$$

Now assume, if the population proportion is not more than 0.3 then $p = 0.3$ and $q = 0.7$. So,

$$\text{Sample Size (n)} = \frac{Z^2 pq}{E^2} = \frac{(1.96)^2\, 0.3(1-0.3)}{(0.03)^2} = \frac{0.806736}{0.0009} = 896.4 \approx 896$$

Example 8: ABC hotel management is interested in determining the percentage of the guests of the hotel who stay for more than 2 days.

The reservation manager wants to be 95% confident that the percentage has been estimated to be within ±3% of the true value, what is the most conservative sample size needed for this problem? ($z = 1.96$ for the given confidence level of 95%)