

Example 17: A random sample of 50 sales invoices was taken from a large population of sales invoices. The average value was found to be ₹2000 with a standard deviation of ₹540. Find a 90 per cent confidence interval for the true mean value of all the sales.

Solution: The information given is: $\bar{X} = 2000$, $s = 540$, $n = 64$, and $\alpha = 10$ per cent.

Therefore,

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{540}{\sqrt{64}} = 67.50 \text{ and } z_{\alpha/2} = 1.64 \text{ (from normal table)}$$

The required confidence interval of population mean μ is given by:

$$\bar{X} \pm z_{\alpha/2} \frac{s}{\sqrt{n}} = 2000 \pm 1.64(67.50) = 2000 \pm 110.70$$

Thus, the mean of the sales invoices for the whole population is likely to fall between ₹1,889.30 and ₹2,110.70, i.e., $1,889.30 \leq \mu \leq 2110.70$.

Example 18: A random sample of 100 observations yields sample mean $\bar{X} = 150$ and sample variance $s^2 = 400$. Compute 95% and 99% confidence interval for the population mean.

Solution: We are given:

$$n = 100, \bar{X} = 150, S^2 = 400 \Rightarrow S = 20$$

$$\text{S.E.}_{\bar{x}} = \frac{S}{\sqrt{n}} = \frac{20}{\sqrt{100}} = 2 \text{ [For large sample, } \sigma = S \text{]}$$

At 95% confidence level, the value of $z_{\alpha/2} = 1.96$

At 99% confidence level, the value of $z_{\alpha/2} = 2.58$

1) 95% confidence interval or limits for μ are:

$$\bar{X} \pm 1.96 \cdot \text{S.E.}_{\bar{x}}$$

Putting the values, we get,

$$150 \pm 1.96 \times 2 = 150 \pm 3.92 = 153.92 \text{ or } 146.08$$

Thus, $146.08 < \mu < 153.92$

2) 99% confidence interval or limits for μ are:

$$\bar{X} \pm 2.58 \cdot \text{S.E.}_{\bar{x}}$$

$$= 150 \pm 2.58 \times 2$$

$$= 150 \pm 5.16$$

$$= 155.16 \text{ or } 144.84$$

Thus, $144.84 < \mu < 155.16$