Solved Problem 4.29

In 30 tosses of a coin the following sequence of heads (H) and tails (T) is obtained

$H\ TT\ H\ T\ H\ HH\ T\ HH\ TT\ H\ T\ H\ T\ HH\ TH\ TT\ HT\ HH\ T\ H\ T$

(a) Determine the number of runs.

(b) Test at the 0.05 significance level whether the sequence is random

Solution

(a) Let us find the number of runs

$\frac{1}{H}\ \frac{2}{TT}\ \frac{3}{H}\ \frac{4}{T}\ \frac{5}{HHH}\ \frac{6}{T}\ \frac{7}{HH}\ \frac{8}{TT}\ \frac{9}{H}\ \frac{10}{T}\ \frac{11}{H}\ \frac{12}{T}\ \frac{13}{HH}\ \frac{14}{T}\ \frac{15}{H}\ \frac{16}{TT}\ \frac{17}{H}\ \frac{18}{T}$

$\frac{19}{HH}\ \frac{20}{T}\ \frac{21}{H}\ \frac{22}{T}$

$\therefore$ The number of runs $R = 22$.

(b) **Null hypothesis** H_0 : The sequence is random.

Alternative hypothesis

H_1 : The sequence is not random.

Level of significance $\alpha = 0.05$

Test statistic

Here $n_1 = 16, n_2 = 14$

$R = 22$

$$\text{Now,} \quad \mu = \frac{2n_1n_2}{n_1+n_2} + 1 = \frac{2 \times 16 \times 14}{16+14} + 1 = 15.93$$

$$\sigma = \sqrt{\frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1+n_2)^2(n_1+n_2-1)}}$$

$$= \sqrt{\frac{2 \times 16 \times 14(2 \times 16 \times 14 - 16 - 14)}{(16+14)^2(16+14-1)}}$$

$$= \sqrt{7.175} = 2.679$$

$$\therefore \quad Z = \frac{R-\mu}{\sigma} \sim N(0,1)$$

$$Z = \frac{22 - 15.93}{2.679} = 2.27$$

The critical value of Z_α at 5% level of significance is 1.96.

Conclusion

Since $2.27 > 1.96$, we reject the null hypothesis H_0 and conclude that the sequence is not random.

Solved Problem 4.30

The production manager of a large undertaking randomly paid 10 visits to the work site in a month. The number of workers who reported late for duty were found to be 2, 4, 5, 1, 6, 3, 2, 1, 7 and 8 respectively. Use the run test for randomness at $\alpha = 0.05$ to check the claim of the production superintendent that on an average not more than 3 workers report late for duty.

Solution

Given

$$\begin{array}{cccccccccc} 2 & 4 & 5 & 1 & 6 & 3 & 2 & 1 & 7 & 8 \\ B & A & A & B & A & — & B & B & A & A \end{array}$$

Here A = the average above 3

B = the average below 3

The above sequence can be written as

$$\underset{1}{\underline{B}} \quad \underset{2}{\underline{AA}} \quad \underset{3}{\underline{B}} \quad \underset{4}{\underline{A}} \quad \underset{5}{\underline{BB}} \quad \underset{6}{\underline{AA}}$$

$\therefore$ $R = 6$ (the number of runs)

$n_1 = 5$ (the number of occurrences of 'A')

$n_2 = 4$ (the number of occurrences of 'B')

H_0 : the sample is randomly chosen

Now, H_1 : the sample is not randomly chosen

α = 0.05

$$\text{Here,} \quad \mu = \frac{2n_1n_2}{n_1 + n_2} + 1$$

$$= \frac{2 \times 5 \times 4}{5 + 4} + 1$$

$$= \frac{40}{9} + 1$$

$$= 4.4444 + 1$$

$$\mu = 5.4444$$

$$\sigma = \sqrt{\frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1+n_2)^2(n_1+n_2-1)}}$$

$$= \sqrt{\frac{2 \times 5 \times 4(2 \times 5 \times 4 - 5 - 4)}{(5+4)^2(5+4-1)}}$$

$$= \sqrt{\frac{1240}{648}}$$

$$\sigma = 1.3833$$

Test statistic

$$\therefore \quad Z = \frac{R-\mu}{\sigma} \sim N(0,1)$$

$$= \frac{6 - 5.4444}{1.3833} = 0.4016$$

The value of Z_α at 5% level of significance is 1.96.

Conclusion

Since $Z < 1.96$, we accept H_0 and we may conclude that the sample is randomly chosen.

Solved Problem 4.31 ***Anna Univ MBA : Jan 2005***

A technician is asked to analyze the results of 22 items made in a preparation run. Each item has been measured and compared to engineering specifications. The order of acceptance 'a' and rejections 'r' is

$$aarrrarraaaaarrarraara$$

Determine whether it is a random sample or not. Use $\alpha = 0.05$.

Solution

Given

$$\underset{1}{\underline{aa}} \quad \underset{2}{\underline{rrr}} \quad \underset{3}{\underline{a}} \quad \underset{4}{\underline{rr}} \quad \underset{5}{\underline{aaaaa}} \quad \underset{6}{\underline{rr}} \quad \underset{7}{\underline{a}} \quad \underset{8}{\underline{rr}} \quad \underset{9}{\underline{aa}} \quad \underset{10}{\underline{r}} \quad \underset{11}{\underline{a}}$$

Here, $n_1 = 12$

$n_2 = 10$

$R = 11$ the number of runs

$$\mu = \frac{2n_1n_2}{n_1+n_2} + 1$$
$$= \frac{2 \times 12 \times 10}{12+10} + 1$$
$$= \frac{240}{22} + 1$$
$$\mu = 11.9091$$

$$\sigma = \sqrt{\frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1+n_2)^2(n_1+n_2-1)}}$$
$$= \sqrt{\frac{2 \times 12 \times 10(2 \times 12 \times 10 - 12 - 10)}{(12+10)^2(12+10-1)}}$$
$$= \sqrt{\frac{52320}{10164}}$$
$$\sigma = 2.2688$$

Now, H_0 : The sample is randomly chosen

H_1 : The sample is not randomly chosen

α = 0.05

Test statistic

$$Z = \frac{R-\mu}{\sigma} \sim N(0,1)$$
$$Z = \frac{11 - 11.9091}{2.2688}$$
$$= -0.4007$$
$$|Z| = |-0.4007| = 0.4007$$

The value of Z_α at $\alpha = 0.05$ level of significance for two tailed test is 1.96.

Conclusion

Since $|Z| < 1.96$, we accept H_0 and conclude that the sample is randomly chosen.

Solved Problem 4.32

In 30 tosses of a coin, the following sequence of head (H) and tails(T) is obtained

$$HTTHTHHHTHHTTHTHTHHTHTTHTHHTHT$$

(a) Determine the number of runs

(b) Test at 0.10 level of significance, whether the sequence is random

Solution

Given

$$\underset{1}{\underline{H}}\ \underset{2}{\underline{TT}}\ \underset{3}{\underline{H}}\ \underset{4}{\underline{T}}\ \underset{5}{\underline{HHH}}\ \underset{6}{\underline{T}}\ \underset{7}{\underline{HH}}\ \underset{8}{\underline{TT}}\ \underset{9}{\underline{H}}\ \underset{10}{\underline{T}}\ \underset{11}{\underline{H}}\ \underset{12}{\underline{T}}$$

$$\underset{13}{\underline{HH}}\ \underset{14}{\underline{T}}\ \underset{15}{\underline{H}}\ \underset{16}{\underline{TT}}\ \underset{17}{\underline{H}}\ \underset{18}{\underline{T}}\ \underset{19}{\underline{HH}}\ \underset{20}{\underline{T}}\ \underset{21}{\underline{H}}\ \underset{22}{\underline{T}}$$

Here, $R = 22$ (the number of runs)

$n_1 = 16$ (the no of occurrences of H)

$n_2 = 14$ (the no of occurrences of T)

$$\mu = \frac{2n_1n_2}{n_1+n_2} + 1$$

$$= \frac{2 \times 16 \times 14}{16+14} + 1$$

$$= \frac{448}{30} + 1$$

$$\mu = 15.9333$$

$$\sigma = \sqrt{\frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1+n_2)^2(n_1+n_2-1)}}$$

$$= \sqrt{\frac{2 \times 16 \times 14(2 \times 16 \times 14 - 16 - 19)}{(16+14)^2(16+14-1)}}$$

$$= \sqrt{\frac{187264}{26100}}$$

$$\sigma = 2.6786$$

Now,

H_0 : The sample is randomly chosen

H_1 : The sample is not randomly chosen

α = 0.10