Test statistic

$$Z = \frac{R-\mu}{\sigma} \sim N(0,1)$$

$$= \frac{22 - 15.9333}{2.6786}$$

$$\therefore \quad Z = 2.2649$$

The value of Z_α at $\alpha = 0.10$ level of significance for two tailed test is 1.645.

Conclusion

Since $Z > 1.645$, we reject our null hypothesis H_0 and we conclude that the sample is not randomly chosen.

Solved Problem 4.33

After a television debate between two political candidates, a telephone line is open to viewers wishing to express their opinions on which the democratic (D) or the republican (R) candidate won the debate. The following sequence represents 19 opinions of viewers in the order in which they telephoned. Using a run test and a significance level of 5% does the sequence indicate a non random order?

$$RRDDRDDRRRRDDRDRDDD$$

Solution

Given

$$\frac{RR}{1}\ \frac{DD}{2}\ \frac{R}{3}\ \frac{DD}{4}\ \frac{RRRR}{5}\ \frac{DD}{6}\ \frac{R}{7}\ \frac{D}{8}\ \frac{R}{9}\ \frac{DDD}{10}$$

Here, $R = 10$ (the number of runs)

$n_1 = 9$ (the number of occurrences of R)

$n_2 = 10$ (the number of occurrences of D)

$$\mu = \frac{2n_1n_2}{n_1+n_2} + 1$$

$$= \frac{2 \times 9 \times 10}{9+10} + 1$$

$$\mu = 10.4737$$

$$\sigma = \sqrt{\frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1+n_2)^2(n_1+n_2-1)}}$$

$$= \sqrt{\frac{2 \times 9 \times 10(2 \times 9 \times 10 - 9 - 10)}{(9+10)^2(9+10-1)}}$$

$$= \sqrt{\frac{28980}{6498}}$$

$$\sigma = 2.118$$

Now,

H_0 : The sample is randomly chosen
H_1 : The sample is not randomly chosen
α = 0.05

Test statistic

$$Z = \frac{R - \mu}{\sigma} \sim N(0,1)$$

$$= \frac{10 - 10.4737}{2.1118}$$

$$= -0.2243$$

$$|Z| = |-0.2243|$$

$$\therefore \quad |Z| = 0.2243$$

The value of Z_α at $\alpha = 0.05$ level of significance for two tailed test is 1.96.

Conclusion

Since $|Z| < 1.96$, we accept H_0 and conclude that the sample is randomly chosen.

4.6 Rank Correlation

Some times we have to deal with problems in which the data cannot be quantitatively measured but qualitative assessment is possible.

Let a group of 'n' individuals be arranged in order of merit in possession of two characteristics A and B. In general, the ranks in the two characteristics are different.

Let (x_i, y_i); $i = 1, 2 \cdots n$ be the ranks of 'n' individuals in the group for two characteristics A and B respectively. The correlation coefficient between the ranks x_i and y_i is called the rank correlation between the two characteristics A and B for that group of individuals.

The Spearman's coefficient of rank correlation is given by

$$\rho_s = 1 - \frac{6\sum_i d_i^2}{n(n^2-1)} \qquad (d_i = x_i - y_i)$$

where d_i^2 is the square of the difference of corresponding ranks and n is the number of pairs of observations.

Note:

1. When the ranks are same, $\rho = 1$
2. The rank correlation co-efficient ρ lies between -1 and 1

   i.e $\quad -1 \leq \rho \leq 1$

4.6.1 Repeated ranks

If there is more than one item with the same value in the series, then common ranks are given to the repeated items. As a result of this, the following adjustment (or) correction is made in the rank correlation formula.

In the rank correction coefficient formula, we add the correction factor $\frac{m(m^2-1)}{12}$ to $\sum d_i^2$, where 'm' is the number of items, an item is repeated. This correction factor is to be added for each repeated value.

$$\therefore \qquad \rho_s = 1 - \frac{6(\sum d_i^2 + \text{correction factor})}{n(n^2-1)}$$

$$\text{i.e} \qquad \rho_s = 1 - \frac{6(\sum d_i^2 + \frac{m(m^2-1)}{12})}{n(n^2-1)}$$

Solved Problem 4.34

The following are the ranks obtained by 10 students in statistics and mathematics. To what extent is knowledge of students in statistics related to knowledge in mathematics?

Statistics :	1	2	3	4	5	6	7	8	9	10
Mathematics :	2	4	1	5	3	9	7	10	6	8

Solution

Rank in statistics (x)	Rank in mathematics (y)	$d = x - y$	d^2
1 7 2	−1	1	
2	4	−2	4
3	1	2	4
4	5	−1	1
5	3	2	4
6	9	−3	9
7	7	0	0
8	10	−2	4
9	6	3	9
10	8	2	4
			$\sum d^2 = 40$

$$\therefore \quad \rho_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 \times 40}{10(100-1)} = 0.76$$

There is high correlation between knowledge in the two subjects.

Solved Problem 4.35

Ten competitors in a beauty contest are ranked by 3 judges in the following order.

A	:	1	6	5	3	10	2	4	9	7	8
B	:	3	5	8	4	7	10	2	1	6	9
C	:	6	4	9	8	1	2	3	10	5	7

Find which pair of judges have the nearest approach to common taste of beauty.

Solution

A	B	C	$d_1 = A - B$	$d_2 = B - C$	$d_3 = A - C$	d_1^2	d_2^2	d_3^2
1	3	6	−2	−3	−5	4	9	2
6	5	4	1	1	2	1	1	4
5	8	9	−3	−1	−4	9	1	1
10	4	8	6	−4	2	36	16	4
3	7	1	−4	6	2	16	36	4
2	10	2	−8	8	0	64	64	0
4	2	3	2	−1	1	4	1	1
9	1	10	8	−9	−1	64	81	1
7	6	5	1	1	2	1	1	4
8	9	7	−1	2	−1	1	4	1
						200	214	60

$\sum d_1^2 = 200, \sum d_2^2 = 214, \sum d_3^2 = 60$

$$\rho_{AB} = 1 - \frac{6 \times 200}{10(100-1)} = -0.212$$

$$\rho_{BC} = 1 - \frac{6 \times 214}{10(100-1)} = -0.297$$

$$\rho_{AC} = 1 - \frac{6 \times 60}{10(100-1)} = 0.636$$

Hence judges A and C have the nearest approach to common tastes of beauty.

Solved Problem 4.36

Calculate Spearman's rank correlation coefficient for the following data.

X :	53	98	95	81	75	71	59	55
Y :	47	25	32	37	30	40	39	45