Panel member	Brand I	Brand II
6	29	21
7	19	23
8	27	22
9	20	23
10	30	20
11	18	18
12	28	21
13	26	17
14	24	26

Solution

Null Hypothesis: $H_0 : p = 0.5$

i.e there is no difference in the level of ratings for the two brands.

Alternative Hypothesis: $H_1 : p \neq 0.5$, (two-tailed test)

i.e there is a difference in the level of ratings for the two brands.

Level of significance: $\alpha = 5\%$

Test Statistic

From the given data,

$$d_i : \; - \; + \; - \; - \; 0 \; - \; + \; - \; + \; - \; 0 \; - \; - \; +$$

Here $n = 4 + 8 = 12$ (by omitting zero differences)

$k =$ number of negative deviations = 8

Now,

$$p' = P(u \geq k) = \left(\frac{1}{2}\right)^n \sum_{x=k}^{n} \binom{n}{x} \qquad (\because \; np < 5)$$

$$= \left(\frac{1}{2}\right)^{12} \sum_{x=8}^{12} \binom{12}{x}$$

$$= \left(\frac{1}{2}\right)^{12} \left[\binom{12}{8} + \binom{12}{9} + \cdots + \binom{12}{10} + \binom{12}{11} + \binom{12}{12}\right]$$

$$= (0.000244)\,(495 + 220 + 66 + 12 + 1)$$

$$= (0.000244)\,(794)$$

$$= 0.1937$$

Conclusion:

Since $p' > 0.05$, (i.e $0.1937 > 0.05$), we accept our null hypothesis H_0 and conclude that there is no significant difference in the level of ratings for the two brands.

Solved Problem 4.4 ***Anna University May/June 2005***

An automotive engineer is investigating 2 different types of metering devices for an electronic fuel injection system to determine whether they differ in their fuel mileage performance. The system is installed on 12 different cars and a test is run with each metering device on each car. The observed fuel mileage performance data are given in the following table. Use the sign test to determine whether the median fuel mileage performance is the same for both devices using 5% level of significance.

Car	:	1	2	3	4	5	6
Device I	:	17.6	19.4	19.5	17.1	15.3	15.9
Device II	:	16.8	20	18.2	16.4	16	15.4

Car	:	7	8	9	10	11	12
Device I	:	16.3	18.4	17.3	19.1	17.8	18.2
Device II	:	16.5	18	16.4	20.1	16.7	17.9

Solution

Null Hypothesis: $H_0 : p = 0.5$

i.e the median fuel mileage performance is the same for both devices.

Alternative Hypothesis: $H_1 : p \neq 0.5$,

i.e the median fuel mileage performance is not the same for both devices.

Level of significance: $\alpha = 5\%$

Test Statistic

From the given data, we have

$$d_i : \; - \; + \; - \; - \; + \; - \; + \; - \; - \; + \; - \; -$$

$\therefore \quad n = 4 + 8 = 12$ and

$k =$ number of negative signs = 8

Now,

$$p' = P(u \geq k) = \left(\frac{1}{2}\right)^n \sum_{x=k}^{n} \binom{n}{x} \qquad (\because \; np < 5)$$

$$= \left(\frac{1}{2}\right)^{12} \sum_{x=8}^{12} \binom{12}{x}$$

$$= \left(\frac{1}{2}\right)^{12} \left[\binom{12}{8} + \binom{12}{9} + \cdots + \binom{12}{10} + \binom{12}{11} + \binom{12}{12}\right]$$

$$= (0.000244)\,(495 + 220 + 66 + 12 + 1)$$

$$= (0.000244)\,(794) = 0.1937$$

Conclusion:

Since $p'\ (0.1937) < 0.05$, we accept our null hypothesis and conclude that the median fuel mileage performance is the same for both devices.

4.3.2 One Sample Sign Test

One-sample sign test is a non-parametric method which is used as an alternate to the one-sample t-test, where the null hypothesis is $\mu = \mu_0$ against a suitable alternative hypothesis.

In the sign test, we replace each sample value exceeding μ_0 with a plus sign and each value less than μ_0 with a minus sign. Then u, the number of plus signs is a value of a random variable having Binomial distribution with parameter n and probability $p = \frac{1}{2}$ (for median). If a sample value equals μ_0, we simply discard it.

In order to performs a sign test, when the sample size is very small, we use Binomial probabilities and when the sample size is large we use the normal approximation to the Binomial distribution with

Mean $= np$ and Variance $= npq$

The test statistic in this case is

$$Z = \frac{u - np}{\sqrt{npq}} \sim N(0,1)$$

Where $p = \frac{1}{2}$ and u, the number of plus signs

Note

For lower quartile $Q_1, p = \frac{1}{4}$

For median quartile $Q_2, p = \frac{1}{2}$

For upper quartile $Q_3, p = \frac{3}{4}$

Solved Problem 4.5

The following are the measurements of breaking strength of a certain kind of 2-inch cotton ribbon in pounds.

163	165	160	189	161	171	158	151	169	162
163	139	172	165	148	166	172	163	187	173

Solution

Null Hypothesis: $H_0 : \mu = 160$

Alternative Hypothesis: $H_1 : \mu > 160$, (one-tailed test)

Level of significance: $\alpha = 5\%$

Test Statistic

Let $u =$ the observed number of plus signs

Replacing each value exceeding 160 with a plus sign, each value less than 160 with a minus sign, and discarding the one value which equals 160, we get

$$+ \; + \; 0 \; + \; + \; + \; - \; - \; + \; + \; + \; - \; + \; + \; - \; + \; + \; + \; + \; +$$

Here $n =$ the total number of plus and minus signs = 19

$u =$ number of plus signs = 15

Now, we find

$$p' = P(u \geq 15) = \left(\frac{1}{2}\right)^n \sum_{x=15}^{n} \binom{19}{x}$$

$$= \left(\frac{1}{2}\right)^{19} \sum_{x=15}^{19} \binom{19}{x}$$

$$= \left(\frac{1}{2}\right)^{19} \left[\binom{19}{15} + \binom{19}{16} + \binom{19}{17} + \binom{19}{18} + \binom{19}{19}\right]$$

$$= 0.0095$$

Conclusion:

Since p' (0.0095), is less than 0.05, we reject our null hypothesis and conclude that the mean breaking strength of given kind of ribbon exceeds 160 pounds.

Solved Problem 4.6 ***Anna University MBA Nov/Dec 2002***

The following data represent the number of hours that a rechargeable hedge trimmer operates before a recharge is required.

1.5, 2.2, 0.9, 1.3, 2.0, 1.6, 1.8, 1.5, 2.0, 1.2 and 1.7 Use the sign test to test the hypothesis of the 0.05 level of significance that this particular trimmer operates with a median of 1.8 hours before requiring a recharge.

Solution

Null Hypothesis: $H_0 : \mu = 1.8$
Alternative Hypothesis: $H_1 : \mu > 1.8$, (two-tailed test)
Level of significance: $\alpha = 0.05$

Test Statistic

Given data is

1.5, 2.2, 0.9, 1.3, 2.0, 1.6, 1.8, 1.5, 2.0, 1.2 and 1.7
Assign '+' if it is greater than 1.8
Assign '–' if it is less than 1.8 and
Assign '0' if it is equal to 1.8, we get

$$- \quad + \quad - \quad - \quad + \quad - \quad 0 \quad - \quad + \quad - \quad -$$

Here $n =$ the total number of plus and minus signs = 10
$u =$ number of plus signs = 3

Now,

$$p' = P(u \geq 3) = \left(\frac{1}{2}\right)^1 0 \sum_{x=3}^{10} \binom{10}{x}$$

$$= \left(\frac{1}{2}\right)^{10} \left[\binom{10}{3} + \binom{10}{4} + \cdots + \binom{10}{10}\right]$$

$$= (0.000976)\,[120 + 210 + 252 + 210 + 120 + 45 + 10 + 1]$$

$$= (0.000976)\,(968)$$

$$= 0.9448$$

Conclusion:

Since p' (0.9448), is greater than 0.05, accept our null hypothesis and conclude that this particular trimmer operates with a median of 1.8 hours before requiring a recharge.

Solved Problem 4.7

The following data in tons, are the amounts of sulphur oxides emitted by a large industrial plant in 40 days.

24	15	20	29	19	18	22	25	27	9
17	20	17	6	24	14	15	23	24	26
19	23	28	19	16	22	24	17	20	13
19	10	23	18	31	13	20	17	24	14

Use the sign test to test the null hypothesis $\mu = 21.5$ against the alternative hypothesis $\mu > 21.5$ at the 0.01 level of significance.

Solution

Null Hypothesis: $H_0 : \mu = 21.5$
Alternative Hypothesis: $H_1 : \mu > 21.5$, (one-tailed test)
Level of significance: $\alpha = 0.01$

Test Statistic

Replacing each value exceeding 21.5 with a plus sign, each value less than 21.5 with a minus sign, we get

$$+ - - + - - + - - + + + - - - - - + - - + + +$$
$$- + + - - + + - - - - - + - + - - - + -$$

Here $u =$ the number of plus signs = 16
$n =$ 16+24=40

As the sample size $n = 40$ is very large, we shall use the normal approximation to Binomial distribution.

$$Z = \frac{u - np}{\sqrt{npq}} \sim N(0,1)$$

Where $p = \frac{1}{2}$ $n = 40$

$$Z = \frac{16 - 40(0.5)}{\sqrt{40(0.5)(0.5)}} = \frac{-4}{3.16} = -1.26$$

$\therefore$ $|Z| = 1.26$.

The critical value Z_α at $\alpha = 0.01$ for one tailed test is 2.33

Conclusion

Since $|Z| < Z_\alpha$ (2.33), we accept the null hypothesis and conclude that $\mu = 21.5$