**Example : 3** ***Two random samples gave the following resuls.***

$$n_1 = 10,\ \Sigma(x-\bar{x})^2 = 90$$

$$n_2 = 12,\ \Sigma(y-\bar{y})^2 = 108$$

Test whether the samples came from the populations with the same variance.

***Solution :***

$H_0 :$ $\sigma_1^2 = \sigma_2^2$ (The samples are drawn from the populations with equal variance)

$H_1 :$ $\sigma_1^2 \neq \sigma_2^2$ (The samples are drawn from the populations with unequal variances)

The variance of the samples are given by

$$S_1^2 = \frac{\Sigma(x_i-\bar{x})^2}{n_1}$$

$$S_1^2 = \frac{90}{10} = 9$$

Variance of the sample B is

$$S_2^2 = \frac{\Sigma(y_i-\bar{y})^2}{n_2}$$

$$S_2^2 = \frac{108}{12} = 9$$

The estimated variances of the two populations

$$S_1^2 = \frac{n_1 S_1^2}{n_1 - 1} = \frac{10 \times 9}{9}$$

$$S_1^2 = 10$$

$$S_2^2 = \frac{n_2 S_2^2}{n_2 - 1} = \frac{12 \times 9}{11}$$

$$S_2^2 = 9.82$$

$$S_1^2 > S_2^2$$

The test statistic is given by

$$F = \frac{S_1^2}{S_2^2} = \frac{10}{9.82} = 1.02$$

$$ndf = (n_1 - 1, n_2 - 1) = (9, 11)$$

Table value of F for (9, 11) df at 5% level = 2.90.

*Conclusion :*

$H_0$ is accepted at 5% level since the calculated value of F < the table value of F.

∴ Variances of two populations are equal.

*Example : 4* *Time taken by workers is performing a job are given below.*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *Method I :* | *20* | *16* | *26* | *27* | *23* | *22* | |
| *Method II:* | *27* | *33* | *42* | *35* | *32* | *34* | *38* |

*Test whether there is any significant difference between the variances of time distribution.*

*Solution :*

Let us first calculate the variance of the samples.

| **Sample I (x – 22)** | | | **Sample II (y –34)** | | |
|---|---|---|---|---|---|
| x | d | $d^2$ | y | d | $d^2$ |
| 20 | –2 | 4 | 27 | –7 | 49 |
| 16 | –6 | 36 | 33 | –1 | 1 |
| 26 | 4 | 16 | 42 | 8 | 64 |
| 27 | 5 | 25 | 35 | 1 | 1 |
| 23 | 1 | 1 | 32 | –2 | 4 |
| 22 | 0 | 0 | 34 | 0 | 0 |
| | | | 38 | 4 | 16 |
| 134 | 2 | 82 | 241 | 3 | 135 |

$$\bar{x} = \frac{134}{6} = 22.33\ ; \qquad \bar{y} = \frac{241}{7} = 34.43$$

$$S_1^2 = \frac{82}{6} - \left(\frac{2}{6}\right)^2 = 13.67 - 0.44 = 13.23$$

$$S_2^2 = \frac{135}{7} - \left(\frac{3}{7}\right)^2 = 19.29 - 0.18 = 19.11$$

$$n_1 = 6; \qquad n_2 = 7$$

$$S_2^2 = 13.23; \quad S_2^2 = 19.11$$

$$S_1^2 = \frac{n_1 S_1^2}{n_1 - 1} = \frac{6 \times 13.23}{5}$$

$$S_1^2 = 15.88$$

$$S_2^2 = \frac{n_2 S_2^2}{n_2 - 1} = \frac{7 \times 19.11}{6}$$

$$S_2^2 = 22.30$$

$$S_2^2 > S_1^2$$

$$H_0 : \sigma_1^2 = \sigma_2^2$$

$$H_1 : \sigma_1^2 \neq \sigma_2^2$$

The test statistic is given by

$$F = \frac{S_2^2}{S_1^2} = \frac{22.30}{15.88}$$

$$F = 1.40$$

$$\text{d.f} = (6, 5)$$

Table value of F at 5% level = 4.28

*Conclusion :*

$H_0$ is accepted since the calculated value of F < the table value of F.

∴ There is no significant difference between the variances of time distribution.

Example : 5 ***Two random samples drawn from normal populations are***

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Sample I :* | *20* | *16* | *26* | *27* | *23* | *22* | *18* | *24* | *25* | *19* | | |
| *Sample II :* | *27* | *33* | *42* | *35* | *32* | *34* | *38* | *28* | *41* | *43* | *30* | *37* |

***Obtain estimates of variances of the population and test whether the two populations have the same variance.***

*Solution :*

Let us first calculate the variance of the samples.

| **Sample I** $x-22$ | | | **Sample II** $y-35$ | | |
|---|---|---|---|---|---|
| x | d | $d^2$ | y | d | $d^2$ |
| 20 | –2 | 4 | 27 | –8 | 64 |
| 16 | –6 | 36 | 33 | –2 | 4 |
| 26 | 4 | 16 | 42 | 7 | 49 |
| 27 | 5 | 25 | 35 | 0 | 0 |
| 23 | 1 | 1 | 32 | –3 | 9 |
| 22 | 0 | 0 | 34 | –1 | 1 |
| 18 | –4 | 16 | 38 | 3 | 9 |
| 24 | 2 | 4 | 28 | –7 | 49 |
| 25 | 3 | 9 | 41 | 6 | 36 |
| 19 | –3 | 9 | 43 | 8 | 64 |
| | | | 30 | –5 | 25 |
| | | | 37 | 2 | 4 |
| 220 | 0 | 120 | 420 | 0 | 314 |

$$\bar{x} = \frac{220}{10} = 22\,; \qquad \bar{y} = \frac{420}{12} = 35$$

$$S_1^{\,2} = \frac{220}{10} = 12$$

$$S_2^{\,2} = \frac{314}{12} = 26.17$$

$$H_0 : \sigma_1^{\,2} = \sigma_2^{\,2}$$

$$H_1 : \sigma_1^{\,2} \neq \sigma_2^{\,2}$$

The estimated variances of the populations are given by

$$S_1^{\,2} = \frac{n_1 S_1^{\,2}}{n_1 - 1} = \frac{10 \times 12}{9} = 13.33$$

$$S_2^{\,2} = \frac{n_2 S_2^{\,2}}{n_2 - 1} = \frac{12 \times 26.17}{11} = 28.55$$

$$S_2^{\,2} > S_1^{\,2}$$

∴ The test statistic is given by

$$F = \frac{S_2^{\,2}}{S_1^{\,2}} = \frac{28.55}{13.33}$$

$$F = 2.14$$

$$\text{d.f} = (11, 9)$$

Table value of F(11, 9) df at 5% level = 3.10

*Conclusion :*

$H_0$ is accepted at 5% level, since the calculated value of F > table value of F.

∴ The variance of the populations are equal.

**Example : 6** ***Two random samples drawn from normal populations are***

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *A* | : | *66* | *67* | *75* | *76* | *82* | *84* | *88* | *90* | *92* | | |
| *B* | : | *64* | *66* | *74* | *78* | *82* | *85* | *87* | *92* | *93* | *95* | *97* |

***Test whether the two populations have the same variance at 5% of significance.***

***Solution :***

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

| Sample I $x - 80$ | | | Sample II $y - 83$ | | |
|---|---|---|---|---|---|
| x | d | $d^2$ | y | d | $d^2$ |
| 66 | −14 | 196 | 64 | −19 | 361 |
| 67 | −13 | 169 | 66 | −17 | 289 |
| 75 | 5 | 25 | 74 | −9 | 81 |
| 76 | −4 | 16 | 78 | −5 | 25 |
| 82 | 2 | 4 | 82 | −1 | 1 |
| 84 | 4 | 16 | 85 | 2 | 4 |
| 88 | 8 | 64 | 87 | 4 | 16 |
| 90 | 10 | 100 | 92 | 9 | 81 |
| 92 | 12 | 144 | 93 | 10 | 100 |
| | | | 95 | 12 | 144 |
| | | | 97 | 14 | 196 |
| 720 | 0 | 734 | 913 | 0 | 1298 |

$\bar{x} = \frac{720}{9} = 80$ ; $\bar{y} = \frac{913}{11} = 83$

$$S_1^2 = \frac{9}{8} \times \frac{734}{9}$$

$$S_1^2 = 91.75$$

$$S_2^2 = \frac{11}{10} \times \frac{1298}{11}$$

$$S_2^2 = 129.8$$

$$S_2^2 > S_1^2$$

∴ The test statistic is given by

$$F = \frac{S_2^2}{S_1^2} = \frac{129.8}{91.75}$$

$$F = 1.41$$

$$d.f = (10, 8)$$

Table value of F = 4.301 at 5% level.

*Conclusion :*

$H_0$ is accepted.

∴ There is no significant difference between the variances of two populations.

**Example : 7** ***Values of a variate in two samples are given below.***

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| *Sample I :* | *5* | *6* | *8* | *1* | *12* | *4* | *3* | *9* | *6* | *10* |
| *Sample II :* | *2* | *3* | *6* | *8* | *1* | *10* | *2* | *8* | | |

***Test the significance of the difference between the two samples means and the two sample variances (or) test the significance the samples came from same normal population.***

***(Hint : If they ask the sample came from normal population to test mean and variance is t and F.)***

***Solution :***

| Sample I | | Sample II | |
|---|---|---|---|
| x | $x^2$ | y | $y^2$ |
| 5 | 25 | 2 | 4 |
| 6 | 36 | 3 | 9 |
| 8 | 64 | 6 | 36 |
| 1 | 1 | 8 | 64 |
| 12 | 144 | 1 | 1 |
| 4 | 16 | 10 | 100 |
| 3 | 9 | 2 | 4 |
| 9 | 81 | 8 | 64 |
| 6 | 36 | | |
| 10 | 100 | | |
| 64 | 512 | 40 | 282 |

$n_1 = 10;$ $n_2 = 8$

$\overline{x} = \frac{64}{10} = 6.4;$ $\overline{y} = \frac{40}{8} = 5.$

$$S_1^2 = \frac{\Sigma x^2}{n_1} - \left(\frac{\Sigma x}{n_1}\right)^2 = \frac{512}{10} - \left(\frac{64}{10}\right)^2$$

$$S_1^2 = 51.2 - 40.96 = 10.24$$

$$S_2^2 = \frac{\Sigma y^2}{n_2} - \left(\frac{\Sigma y}{n_2}\right)^2 = \frac{282}{8} - \left(\frac{40}{8}\right)^2$$

$$S_2^2 = 35.25 - 25 = 10.25$$

*Case - I : Test for Mean*

The samples are small and so we apply t – test.

$H_0$ : $\mu_1 = \mu_2$ (There is no significant difference beween the means of the two populations from which these samples are drawn).

$H_1$ : $\mu_1 \neq \mu_2$ (There is significant difference beween the means of the two populations from which these samples are drawn).

$$S^2 = \frac{n_1 S_1^{\,2} + n_2 S_2^{\,2}}{n_1 + n_2 - 1}$$

$$= \frac{10 \times 10.24 + 8 \times 10.25}{10 + 8 - 2}$$

$$= \frac{184.4}{16} = 11.525$$

$$S = \sqrt{69.125}$$

$$S = 3.395$$

The test statistic is

$$t = \frac{\bar{x} - \bar{y}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{6.4 - 5}{3.395\sqrt{\frac{1}{10} + \frac{1}{8}}}$$

$$t = \frac{1.4}{1.610} = 0.87$$

$$d.f = n_1 + n_2 - 2 = 16$$

Table value of t for 16 df at 5% level = 2.12.

*Conclusion :*

$H_0$ is accepted at 5% level since the calculated value of t < the table value of t.

∴ The means of the populations are equal.

*Case - II : Test for Variance*

$n_1 = 10; \qquad n_2 = 8$

$S_1^2 = 10.24; \qquad S_2^2 = 10.25$

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

$$S_1^2 = \frac{n_1 S_1^2}{n_1 - 1} = \frac{10 \times 10.24}{9}$$

$S_1^2 = 11.38$

$$S_2^2 = \frac{n_2 S_2^2}{n_2 - 1} = \frac{8 \times 10.25}{7}$$

$S_2^2 = 11.71$

Notice that here $S_1^2 > S_2^2$ and so we take the test statistic is

$$F = \frac{S_2^2}{S_1^2} = \frac{11.71}{11.38} = 1.03$$

$$d.f = ((n_2 - 1), (n_1 - 1)) = (7, 9)$$

Table value of F = 3.29.

*Conclusion :*

$H_0$ is accepted at 5% level since the calculated value of F < the table value of F.

∴ The variances of the population are equal.

## 3.4 TEST OF SIGNIFICANCE FOR PROPORTIONS USING NORMAL DISTRIBUTION

### Test for a Single Proportion

Suppose a large sample of size n its taken from a normal population to test the significance difference between a sample population Q and the population proportion P we use statistic.

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

### PROBLEMS UNDER PROPORTIONS AND DIFFERENCES

**Example : 1** ***In a sample of 1,000 people in Maharashtra, 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat are equally popular in this state at 1% level of significance?***

*Solution :*

*Null Hypothesis $H_0$ :*

Both rice and wheat eaters are equally popular.

$$n = 1000, \qquad x = 540$$

$$p = \frac{x}{n} = \frac{540}{1000} = 0.54.$$

$$P = 0.5$$

We know that,

$$P + Q = 1$$

$$Q = 1 - P$$

$$= 1 - 0.5$$

$$Q = \tfrac{1}{2}.$$

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.54 - 0.50}{\sqrt{\frac{(0.5)(0.5)}{1000}}}$$

$$= \frac{0.04}{0.0138} = 2.532$$

$$Z = 2.532$$

Table value

= n degree of freedom at 1%.

= 2.58

*Conclusion :*

Calculated value < Table value

$2.532 < 2.58.$

We accept $H_0$. We may conclude that rice and wheat eaters are equally popular in Maharashtra State.

**Example : 2** ***In a big City 325 men out of 600 men were found to be smokers. Does this information support the conclusion that the majority of men in this city are smokers?***

*Solution :*

*Null Hypothesis $H_0$ :*

Number of smokers = Non smokers in the City.

*Given :*

$n = 600, \quad x = 325, \quad P = ?$

$$p = \frac{x}{n} = \frac{325}{600} = 0.5417.$$

P = Population proportion of smokers in City = 0.5

We know that,

$$P + Q = 1$$

$$Q = 1 - P$$

$$= 1 - 0.5$$

$$Q = 0.5.$$

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

$$= \frac{0.5417 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{600}}}$$

$$= \frac{0.0417}{0.0204} = 2.04$$

$$Z = 2.04$$

Table value

= n degree of freedom at 5%.

= 1.96

***Conclusion :***

Calculated value > Table value

2.04 > 1.96.

We reject $H_0$ at 5% level of significance. The number of smokers and nonsmokers are not equal in the city.

**Example : 3** ***A coin is tossed 144 times and a person gets 80 heads. Can we say that the coin is unbiased one?***

***Solution :***

***Null Hypothesis $H_0$ :***

The coin is unbiased.

***Given :***

n = 144, p = 80.

P = Probability of getting a head in a toss = ½.

We know that,

$$P = \frac{1}{2}$$

$$P + Q = 1$$

$$Q = 1 - P$$

$$= 1 - \frac{1}{2}$$

$$Q = 0.5.$$

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

$$= \frac{0.55 - 0.50}{\sqrt{\frac{0.5 \times 0.5}{144}}} = \frac{0.055}{0.0417}$$

$$Z = 1.31$$

Table value

= n degree of freedom at 5% level of significance.

= 1.96

*Conclusion :*

Calculated value < Table value

$1.31 < 1.96.$

We accepted $H_0$. The coin is unbiased.

**Example : 4** ***A random sample of 500 toys was taken from a large consignment and 65 were found to be defective. Find the percentage of different toys in the consignment?***

*Solution :*

*Null Hypothesis $H_0$ :*

The toys are defective.

*Given :*

$$n = 500, \qquad x = 65$$

$$P = \frac{65}{500} = 0.13$$

We know that,

$$P + Q = 1$$

$$Q = 1 - P$$

$$= 1 - 0.13$$

$$Q = 0.87$$

The limits for population P are given by

$$= P \pm 3\sqrt{\frac{PQ}{n}}$$

$$= 0.13 \pm 3\sqrt{\frac{0.13 \times 0.87}{500}}$$

$$= 0.13 \pm 3(0.015)$$

$$= 0.13 \pm 0.045$$

$$= 0.175 \text{ and } 0.085$$

$$= 17.5\% \text{ and } 8.5\%$$

## EXERCISE

1. A person threw 10 dice 500 times and obtained 2560 times 4, 5 ot 6. Can this be attributed to fluctuations in sampling?

   *Ans : Z = 1.697*

2. A coin is tested 400 times and it turns up head 216 times. Discuss whether the coin is an unbiased one?

   *Ans : The coin is unbiased.*

3. In a sample of 500 people in Kerala 280 are tea drinkers and the rest are coffee drinkers. Can we assume that both coffee and tea are equally popular in the state at 5% level of significance?

*Ans : Z = 2.68.*

4. In a hospital 475 female and 525 male babies were born in a week. Do these figures confirm the hypothesis that males and females are equal in numbers?

***Ans : We may accept the null hypothesis.***

### 3.5.1 TESTING OF SIGNIFICANT OF DIFFERENCE OF PROPORTIONS OF TWO SAMPLES

Suppose two large samples of sizes $n_1$ and $n_2$ respectively. From two different populations to test the significance of difference between the 2 samples t and T.

Then the test statistics is

$$Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where

$$P = \frac{n_1P_1 + n_2P_2}{n_1 + n_2}$$

$$Q = 1 - P$$

### PROBLEMS UNDER DIFFERENCE OF PROPORTIONS

***Example : 1*** ***Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportions of men and infavour of the proposal are same against that they are not at 5% level.***

***Solution :***

***Null Hypothesis $H_0$ :***

$P_1 = P_2$ [there is no significant difference]

*Given :*

$$n_1 = 400, \qquad x_1 = 200$$

$$n_2 = 600, \qquad x_2 = 325$$

$$P_1 = \frac{x_1}{n_1} = \frac{200}{400} = 0.5$$

$$P_2 = \frac{x_2}{n_2} = \frac{325}{600} = 0.541$$

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2}$$

$$= \frac{400 \times 0.5 + 600 \times 0.541}{400 + 600}$$

$$P = 0.525$$

We know that,

$$P + Q = 1$$

$$Q = 1 - P$$

$$= 1 - 0.525$$

$$Q = 0.475$$

$$|Z| = \frac{0.500 - 0.541}{\sqrt{(0.525)(0.475)\left(\frac{1}{400} + \frac{1}{600}\right)}}$$

$$= \frac{-0.041}{\sqrt{0.001039}}$$

$$= -1.269$$

$$|Z| = 1.269$$

Table value

= n degrees of freedom at 5%.

= 1.96

***Conclusion :***

Calculated value < Table value

1.269 < 1.96

We accept $H_0$.

Men and women do not differ significantly as regards proposal of flyover is concerned.

**Example : 2** ***A machine produced 20 defective units in a sample of 400. After over handling the machine, if produced 10 defective units in a batch of 300. Has the machine improved in production due to over handling. Test it as 5% level of significance?***

***Solution :***

***Null Hypothesis $H_0$ :***

$P_1 = P_2$ [there is no significant difference]

***Given :***

$n_1 = 400, \qquad x_1 = 20$

$n_2 = 300, \qquad x_2 = 10$

$$P_1 = \frac{x_1}{n_1} = \frac{20}{400} = \frac{1}{20} = 0.05$$

$$P_2 = \frac{x_2}{n_2} = \frac{10}{300} = \frac{1}{30} = 0.033$$

$$P = \frac{n_1P_1 + n_2P_2}{n_1 + n_2}$$

$$= \frac{400 \times 0.05 + 300 \times 0.033}{400 + 300}$$

$$= \frac{20 + 9.9}{700} = \frac{29.9}{700} = 0.0427$$

$$P + Q = 1$$

$$Q = 1 - P$$

$$= 1 - 0.0427$$

$$Q = 0.9573$$

$$Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$|Z| = \frac{0.050 - 0.033}{\sqrt{0.0427 \times 0.9573\left(\frac{1}{400} + \frac{1}{600}\right)}}$$

$$= \frac{0.017}{\sqrt{0.01545}}$$

$$|Z| = 1.1$$

Table value

$= 1.96$

***Conclusion :***

Calculated value < Table value

$1.1 < 1.96$

We accept $H_0$.

The machine has not improved after over hauling.

**Example : 3** ***In two large populations there are 30 and 25 percent respectively of blue-eyed people. Is this difference likely to be hidden in samples of 1200 and 900 respectively from the two populations?***

*Solution :*

*Null Hypothesis $H_0$ :*

$P_1 = P_2$ [the sample proportions are equal]

*Given :*

$$n_1 = 1200, \qquad n_2 = 900$$

$$P_1 = 30\% = \frac{30}{100} = 0.30$$

$$P_2 = 25\% = \frac{25}{100} = 0.25$$

$$P = \frac{n_1P_1 + n_2P_2}{n_1 + n_2}$$

$$= \frac{1200 \times 0.30 + 900 \times 0.25}{1200 + 900}$$

$$= 0.2785$$

$$P + Q = 1$$

$$Q = 1 - P$$

$$= 1 - 0.2785$$

$$Q = 0.7214$$

$$Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$|Z| = \frac{0.30 - 0.25}{\sqrt{0.2785 \times 0.7214\left(\frac{1}{1200} + \frac{1}{900}\right)}}$$

$$= \frac{0.0500}{\sqrt{0.0004}}$$

$$= \frac{0.0500}{0.0198} = 2.53$$

$$|Z| = 2.53$$

Table value

$= 1.96$

*Conclusion :*

Calculated value > Table value

$2.53 > 1.96$

We reject $H_0$.

The sample propulsions are not equal.

**Example : 4** ***In a rural area were no development of undertaken 160 out of a sample of 250 farmers were indebited. In another area were development work in progress 84 out of sample of 150 farmers were indebited compute the two sample for the following data ?***

*Solution :*

***Null Hypothesis $H_0$ :***

$P_1 = P_2$ [there is no significant difference]

*Given :*

$n_1 = 250, \quad x_1 = 160$

$n_2 = 150, \quad x_2 = 84$

$$P_1 = \frac{x_1}{n_1} = \frac{160}{250} = 0.64$$

$$P_2 = \frac{x_2}{n_2} = \frac{84}{150} = 0.56$$

$$P = \frac{n_1P_1 + n_2P_2}{n_1 + n_2}$$

$$= \frac{250 \times 0.64 + 150 \times 0.56}{250 + 150}$$

$$= \frac{160 + 84}{400}$$

$$= 0.61$$

$$P + Q = 1$$

$$Q = 1 - 0.61$$

$$Q = 0.39$$

$$Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$|Z| = \frac{0.64 - 0.56}{\sqrt{0.61 \times 0.39\left(\frac{1}{250} + \frac{1}{150}\right)}}$$

$$|Z| = 1.53$$

Table value : 1.96

***Conclusion :***

Calculated value < Table value

$1.53 < 1.96$

We accept $H_0$.

The is no significant difference.

## 3.6 LARGE SAMPLE 't' TEST (n > 30)

**Test of Significance For Single Mean :**

Suppose we want to test whether the given sample of size n has been drawn from a population with mean $\mu$. We set up null hypothesis that there is no difference between $x$ and $\mu$.

The test statistic is

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim \text{n degrees of freedom}$$

where

$\bar{x} \rightarrow$ Sample mean

$\mu \rightarrow$ Population mean

$\sigma \rightarrow$ Standard deviation

$n \rightarrow$ Sample size

If the population S.D is not known then use the static.

$$Z = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}}$$

where

$S \rightarrow$ Sample deviation

**Note** The values $x \pm 1.96 \frac{\sigma}{\sqrt{n}}$ are called 95% fiducial limits.

Similarly $x \pm 2.58 \frac{\sigma}{\sqrt{n}}$ are called 99% confidence limits.

## PROBLEMS BASED ON LARGE SAMPLE 't' TEST

**Example : 1** ***The average marks in mathematics of a sample of 100 students was 51 if a standard deviation of 6 would this have been a random sample in a population with average marks 50?***

*Solution :*

*Null Hypothesis $H_0$ :*

The sample is drawn from a population with mean $\mu = 50$.

*Alternative Hypothesis $H_1$ :*

$\mu \neq 50$

*Given :*

$n = 100$

$\overline{x} = 51$

$\mu = 50$

$S \text{ (or) } \sigma = 6$

$$Z = \frac{51 - 50}{\frac{6}{10}}$$

$$= \frac{1}{\frac{6}{10}} = \frac{10}{6}$$

$$Z = 1.66$$

Table value :

= n degrees of freedom at 5%.

= 1.96

*Conclusion :*

Calculated value < Table value

We accept $H_0$.

The sample is drawn from a population with mean $\mu = 50$.

**Example : 2** *A sample of 900 members has a mean of 3.4 cms and S.D 2.61 cms. Is the sample from a large population of mean 3.25 cm and S.D 2.61 cms if the population is normal and its mean is unknown. Find the 95% fiducial limits of true mean?*

***Solution :***

***Null Hypothesis $H_0$ :***

The sample is drawn from a population with mean $\mu = 3.25$.

*Given :*

$n = 900$

$\bar{x} = 3.4$ cm

$\mu = 3.25$ cm

S (or) $\sigma = 2.61$

$$Z = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}}$$

$$= \frac{3.4 - 3.25}{\frac{2.61}{\sqrt{900}}}$$

$Z = 1.724$

Table value :

= n degrees of freedom at 5%.

= 1.96

***Conclusion :***

Calculated value < Table value

$1.724 < 1.96$

We accepted the $H_0$.

The sample is drawn from a population with mean $\mu = 3.25$.

95% confidence limits are

$$= x \pm 1.96 \frac{\sigma}{\sqrt{n}}$$

$$= 3.4 \pm 1.96 \times \frac{2.61}{\sqrt{900}}$$

$$= 3.4 \pm 0.1705$$

$$= 3.57 \text{ and } 3.22955$$

**Example : 3** ***A test was given to a large group of boys whose scored on the average 64.5 marks. The same test was given to a group of 400 boys who score on the average 62.5 boys with a standard deviation 12.5 marks. To test the mean for the above data.***

*Solution :*

*Null Hypothesis $H_0$ :*

The sample is drawn from a population with mean $\mu = 64.5$.

*Given :*

$n = 400$

$\bar{x} = 62.5$ cm

$\mu = 64.5$ cm

$S$ (or) $\sigma = 12.5$

$$|Z| = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}}$$

$$= \frac{62.5 - 64.5}{\frac{12.5}{\sqrt{400}}} = \frac{-2}{\frac{12.5}{20}}$$

$$Z = -3.2$$

$$|Z| = 3.2$$

Table value :

= n degrees of freedom at 5%.

= 1.96

*Conclusion :*

Calculated value > Table value

3.2 < 1.96

We rejected the $H_0$.

The sample is not drawn from a population with mean $\mu \neq 64.5$.

## 3.6.1 TEST OF SIGNIFICANCE FOR DIFFERENCE OF MEAN

Let $\bar{x}_1$ be the mean of a sample of size $n_1$, from a population with mean $\mu_1$ and variance $\sigma_1^2$.

Let $\bar{x}_2$ be the mean of a sample of size $n_2$ from a population with mean $\mu_2$ and variance $\sigma_2^2$.

To test whether there is any significant difference between $\bar{x}_1$ and $\bar{x}_2$ we have to use the statistic.

$$|Z| = \frac{\bar{x}_1 - \bar{x}_1}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim \text{n degrees of freedom}$$

## PROBLEMS BASED ON DIFFERENCE OF MEAN

*Example : 1* ***The means of 2 large samples 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples are regarded as drawn from the same population of S.D 2.5 inches?***

*Solution :*

*Null Hypothesis $H_0$ :*

The samples have been drawn from the same population of S.D 2.5 inches.

i.e., $\mu_1 = \mu_2$ and $\sigma = 2.5$ inches.

$$|Z| = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{67.5 - 68}{\sqrt{\frac{2.5^2}{1000} + \frac{2.5^2}{2000}}}$$

$$= \frac{-0.5}{0.0968}$$

$$= -5.16$$

$$|Z| = -5.16$$

Table value :

= n degrees of freedom at 5%.

= 1.96

***Conclusion :***

Calculated value > Table value

5.16 > 1.96

We rejected the $H_0$.

The samples are not drawn from the same population of S.D 2.5 inches.

*Example : 2* ***A sample of heights are 6400 English men has a mean of 170 cm and a standard deviation of 6.4 cm while a sample of heights of while a sample of heights of 1600 Americans has a mean of 172 cm of a S.D of 6.3 cm. Do the data indicate that the American average taller than the Englishman?***

***Solution :***

***Null Hypothesis $H_0$ :***

Both means do not differ significantly.

$n_1 = 6400$ $\quad\quad$ $n_2 = 1600$

$\bar{x}_1 = 170$ $\quad\quad$ $\bar{x}_2 = 172$

$\sigma_1 = 6.4$ $\quad\quad$ $\sigma_2 = 6.3$

$$|Z| = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{170 - 172}{\sqrt{\frac{6.4^2}{6400} + \frac{6.3^2}{1600}}}$$

$$|Z| = 11.49$$

Table value :

= n degrees of freedom at 5%.

= 1.96

*Conclusion :*

Calculated value > Table value

11.49 > 1.96

We rejected the $H_0$.

**Example : 3** ***The mean yield of wheat from a district A was 210 pounds with standard deviation 10 pounds per acre from a sample of 100 plots. In another district the mean yield was 220 pound with standard deviation 12 pound from a sample of 150 plots. Test whether is any significant difference between crops in the two districts?***

*Solution :*

There is no significant difference between two samples.

$n_1 = 100$ $n_2 = 150$

$\bar{x}_1 = 210$ $\bar{x}_2 = 220$

$\sigma_1 = 10$ $\sigma_2 = 12$

$$|Z| = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{210 - 220}{\sqrt{\frac{10^2}{100} + \frac{12^2}{150}}}$$

$$|Z| = 7.14$$

Table value :

= n degrees of freedom at 5%.

= 1.96

***Conclusion :***

Calculated value > Table value

7.14 > 1.96

We accepted the $H_0$.